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Some issues in non-commutative tachyon condensation

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Abstract: Techniques of non-commutative field theories have proven to be useful in describing D-branes as tachyonic solitons in open string theory. However, this procedure also leads to unwanted degeneracy of solutions not present in the spectrum of D-branes in string theories. In this paper we explore the possibility that this apparent multiplicity of solutions is due to the wrong choice of variables in describing the solutions, and that with the correct choice of variables the unwanted degeneracy disappears.

Keywords: Bosonic Strings, D-branes.

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Contents

1. Introduction and summary

It has been conjectured that the tachyonic vacuum in open bosonic string theory on a D-brane describes the closed string vacuum without D-branes, and that various soliton solutions in this theory describe D-branes of lower dimension [1]. Similar conjectures have also been put forward for superstring theories [2, 3, 4]. Evidence for these conjectures come from both, first [5, 6] and second [7] quantized string theories.

Recently it has been realized that the study of these conjectures can be simplified by examining D-branes in the background of anti-symmetric tensor fields, or equivalently, in the presence of background magnetic field on the D-brane world-volume. In this case the world-volume theory on the D-brane can be described by a non-commutative field theory [8]–[11]. In particular, the non-commutative solitons discussed in [12] can be used to construct exact solutions of the field equations in the limit of infinite background magnetic field [13]–[19], which can then be identified to lower-dimensional D-branes. This reproduces the correct tension of the lower-dimensional D-branes, and also reproduces many of the known features of the world-volume theory of the D-branes.

One of the shortcomings of the analysis of refs. [13, 14] is that it requires infinite background magnetic field, whereas the conjectures involving tachyon condensation are expected to hold for arbitrary values of the background magnetic field including zero background magnetic field. In a recent paper [20] (see also [21]) it was suggested that even if we start with zero or finite magnetic field background, in the tachyonic vacuum the magnetic field dynamically rolls down to infinite value. This proposal, if correct, would provide an exact description of tachyonic soliton solution in all cases. However, as was found in ref. [20], this proposal suffers from the problem that besides the soliton solutions representing lower-dimensional D-branes, there are many other degenerate solutions which do not have any obvious physical interpretation. Furthermore there is no obvious dynamical mechanism which makes the magnetic field roll down to infinity, since different vacua labelled by different values of the magnetic field all appear to have the same energy density. Additional degeneracies in the set of solutions was discussed in ref. [22].

In fact, the problem of unwanted degeneracy appears even in the absence of any background field strength. To see this, let us consider a Dp-brane in the bosonic string theory in 26-dimensional flat space-time. Before tachyon condensation, the world-volume theory of the Dp-brane contains $(25-p)$ massless scalar fields representing coordinates of the brane transverse to its world-volume. These fields can be regarded as the Goldstone modes associated with spontaneously broken translational symmetry in the $(25-p)$ transverse directions. Since the tachyonic vacuum is conjectured to represent the closed string vacuum without any D-brane, we should expect that full $(25+1)$-dimensional translational symmetry is restored in this vacuum. Hence the Goldstone modes, and consequently the flat directions in the potential in the D-brane world-volume theory, should disappear in this vacuum. However, in terms of the variables used in describing the Born-Infeld action, the flat directions in the potential continue to persist in this vacuum, thereby giving us unwanted degeneracy of the vacuum.

In this note we propose a resolution of the above problem, as well as the degeneracy problem encountered in ref. [20]. We propose that the apparent multiplicity of the ground state, labelled by different values of the background magnetic field and/or different positions of the initial D-brane, is due to the fact that the original choice of field variables, used in describing the Born-Infeld action on the D-brane, becomes singular at the tachyonic vacuum, and that with the correct choice of field variables, all these apparently different vacua correspond to the same point in the configuration space. Besides removing the vacuum degeneracy, this proposal also resolves the problem encountered in ref. [20] of unwanted soliton solutions, since many apparantly different solutions in the original variables represent the same field configuration in the new set of variables.

It is best to illustrate this by drawing an analogy. Consider, for example a particle moving in three dimensions under the influence of a potential which has a unique minimum at the origin, and let us suppose further that the potential is symmetric under a rotation about the z-axis. In this case, if we use the spherical polar coordinates (r, θ, ϕ) to describe the motion of the particle, then it will appear that there are infinite number of degenerate ground states of the system, corresponding to $r = 0$, (θ, ϕ) arbitrary. Furthermore, it will not be manifest that the ground state is invariant under the rotational symmetry about the z-axis; since under this transformation ϕ will transform to $\phi + a$ for some constant a. Only after going to a non-singular coordinate system (e.g. the cartesian coordinate system) we see that the ground state is unique and that it is invariant under rotation about the z-axis.

We argue that the case at hand, — the dynamics of the D-brane around the tachyonic vacuum — is similar to the example discussed above; and that the usual variables appearing in the (non-commutative) Born-Infeld action correspond to a singular coordinate system around the tachyonic vacuum. The role of the angular variables (θ, ϕ) is played by the background gauge and massless scalar fields, and the role of the radial variable r is played by $T - T_{\rm min}$, where T denotes the tachyon field and $T_{\rm min}$ is its vacuum value where the potential has a local minimum. Thus at $T = T_{\rm min}$, different values of background gauge and massless scalar fields describe the same configuration. In the case of the three-dimensional particle, the singularity of the spherical polar coordinate system near the origin becomes apparent if we examine the kinetic term of the particle in this coordinate system; as we shall see, the same situation holds in the case of D-branes. (Unfortunately, however, here not all the relevant terms are known which allows us to determine precisely the right choice of coordinate system around the new vacuum.)

We can push the analogy a bit further by comparing deformation of the azimuthal angle ϕ to the deformation of field configurations on the D-brane generated by various symmetry transformations, and the deformation of the polar angle θ to deformation of background magnetic field strength on the D-brane. The analog of rotational symmetry restoration at $r = 0$ will then be the restoration of translation symmetry in directions transverse to the D-brane, and also the restoration of the $\mathrm{U}(\infty)$ gauge symmetry discussed in refs. [14, 20] in the tachyonic vacuum. For a non-central potential which is invariant under rotation about the z-axis, different values of θ describe inequivalent configurations, but at $r = 0$ they all correspond to the same configuration. Similarly, for the D-brane system, we shall argue that although different background magnetic field configurations describe inequivalent configurations away from the tachyonic vacuum, at the tachyonic vacuum they all correspond to the same configuration. Finally, just as in the case of a three-dimensional point particle the rotational symmetry about the z-axis is restored for general r at special values of θ, namely 0 and π, so in the case of a D-brane the $\mathrm{U}(\infty)$ symmetry is restored for a general space-time independent tachyonic background at special value of the magnetic field strength, namely ∞ [14, 20].

Incidentally, we would like to mention here that string field theory [23], whose variables are related to the ones appearing in the Born-Infeld action via a non-trivial field redefinition [24, 25], automatically chooses the right set of field variables around the tachyonic vacuum. This is seen by noting the absence of Goldstone modes, i.e. flat directions of the potential around the new vacuum [26, 27, 28]. Indeed, most of the hard evidence for the proposal put forward in this paper comes from this result in string field theory.

Besides the existence of unwanted solutions, another problem that the results of refs. [14, 20] suffered from was the existence of unwanted open string states on the world-volume of the tachyonic soliton. When we interpret the tachyonic soliton as a D-brane, then these states correspond to open string states with one end on the D-brane represented by the soliton, and the other end in the vacuum. Since open strings cannot end on the vacuum, the spectrum *should not* contain such states; hence existence of these states on the soliton world-volume poses a problem. We propose a resolution of this problem by noting that the equations of motion of U(1) gauge field living on the brane before tachyon condensation will force the currents associated with such states to vanish, and hence these states cannot exist in isolation. (This follows the suggestion in ref. [29], alternative but similar proposals have been made in refs. [30, 31].)

The rest of the paper is organised as follows. In section 2 we discuss how translation invariance in directions transverse to the brane is restored at the tachyonic vacuum. In section 3 we show how the $U(\infty)$ gauge invariance can be restored at the tachyonic vacuum even when the background magnetic field is finite. The field redefinition required for achieving this also removes the unwanted degeneracy of vacuum and soliton solutions. In section 4 we discuss how the unwanted states living on the soliton world-volume, corresponding to open strings with one end on the soliton and the other end in the tachyonic vacuum, might be removed from the spectrum.

We end this section by noting that although we shall carry out our discussion in the context of D-branes in bosonic string theory, we expect that an identical analysis can be carried out for D-branes in superstring theory as well. In particular, restoration of the full supersymmetry at the tachyonic vacuum should follow in a manner analogous to the restoration of the translation invariance discussed in section 2.

2. Restoration of translation invariance at the tachyonic vacuum

Let us consider a Dp-brane in 26-dimensional bosonic string theory in flat Minkowski space-time. This system has $(25-p)$ massless scalar fields living on its world-volume, describing transverse motion of the Dp-brane. These can be regarded as the Goldstone bosons associated with the spontaneously broken translational invariance along these $(25-p)$ directions in the presence of the brane. Now when the tachyon condenses into its ground state, we expect that the translational symmetry of the configuration should be restored fully, and hence these Goldstone modes should disappear. The question is: can we see this in the effective field theory describing the system?

Let us consider the Born-Infeld action describing the system in the static gauge [29, 32]:

$$S_{\mathrm{BI}} \propto \int d^{p+1}x\, \mathcal{V}(T)\sqrt{\det\left(\eta_{\mu\nu} + \partial_\mu\chi^i\partial_\nu\chi^i\right)} + \cdots\,, \tag{2.1}$$

where χ^i denote the transverse coordinates of the brane $(p+1 \leq i \leq 25)$, x^μ denote the world-volume coordinates of the brane $(0 \leq \mu \leq p)$, T denotes the tachyon field, and $\mathcal{V}(T)$ denotes the tachyon potential with the D-brane tension term included, so that $\mathcal{V}(T)$ vanishes at the tachyonic vacuum $T = T_{\rm min}$. For simplicity we have ignored the gauge field terms in (2.1). $\cdots$ denotes terms containing derivatives of T and $\partial_\mu \chi^i$. The action (2.1) has manifest translational invariance along the transverse directions:

$$\delta \chi^i = a^i \,. \tag{2.2}$$

Note, however that this symmetry is spontaneously broken, as is indicated by the presence of field independent terms on the r.h.s. of eq. (2.2). These do not vanish even when all the fields are set to zero.

Now consider the tachyonic vacuum $T = T_{\rm min}$. The r.h.s. of eq. (2.2) is independent of T, and so continues to be non-zero even in this vacuum. Thus it would seem that the vacuum is still infinitely degenerate, labelled by different values of $\langle \chi^i \rangle$, and that the χ^i's continue to represent the Goldstone modes associated with broken translation invariance. However, since $\mathcal{V}(T_{\rm min}) = 0$, the kinetic term of χ^i in (2.1) vanishes at $T = T_{\rm min}$. Thus the fields χ^i are not good coordinates for describing the field configuration around the vacuum $T = T_{\rm min}$. Unfortunately, due to our limited knowledge of the structure of the effective action around the tachyonic vacuum, we are unable to determine the precise expressions for the "good" variables in terms of T and χ^i. However, since the kinetic term of χ^i vanishes at $T = T_{\rm min}$, we would expect that the good choice of field variables, $\widetilde{T}$ and $\widetilde{\chi}^i$, should be related to T and χ^i by functional relations of the form:

$$\widetilde{\chi}^i = h^i(T, \vec{\chi}) \,, \qquad \widetilde{T} = h(T, \vec{\chi}) \,, \tag{2.3}$$

with $h^i(T, \vec{\chi})$ and $h(T, \vec{\chi})$ having the property that[1]

$$h^i(T_{\rm min}, \vec{\chi}) = 0 \,, \qquad h(T_{\rm min}, \vec{\chi}) = \widetilde{T}_{\rm min} \,. \tag{2.4}$$

Here $\widetilde{T}_{\rm min}$ is a constant which can be set to zero by suitably redefining $\widetilde{T}$. In these variables eq. (2.2) takes the form:

$$\delta \widetilde{\chi}^i = \frac{\partial h^i(T, \vec{\chi})}{\partial \chi^j} a^j \,, \qquad \delta \widetilde{T} = \frac{\partial h(T, \vec{\chi})}{\partial \chi^j} a^j \,. \tag{2.5}$$

Since eq. (2.4) is satisfied for all $\vec{\chi}$, we see that $\delta \widetilde{\chi}^i$ and $\delta \widetilde{T}$ vanish at $T = T_{\rm min}$. Thus translation invariance along directions transverse to the D-brane is restored around this vacuum.

As has already been noted in the introduction, string field theory automatically chooses the right coordinate system around the vacuum $T = T_{\rm min}$, since the string field theory potential expanded around $T = T_{\rm min}$ has no flat direction [26, 27]. Indeed, this result of string field theory can be turned around to conclude that the correct choice of coordinate system around $T = T_{\rm min}$ must be of the form given in eqs. (2.3) and (2.4) so that in this coordinate system there are no flat directions of the potential.

[1] For our purpose it will be sufficient if these relations hold for constant $\vec{\chi}$ configurations.

3. Lifting of degeneracy of solutions

In the analysis of ref. [20] the authors proposed a specific form of the tachyonic soliton solution on a D-brane, representing a D-brane of lower dimension, but also noted that there are other solutions with no obvious interpretation as D-branes. In this section we shall argue that all these are equivalent solutions, in the same sense that different values of θ and ϕ for $r = 0$ represent the same configuration of a particle moving in 3-dimensions.

The configuration we study is the same one studied in ref. [20], — a D25-brane in the presence of constant background antisymmetric tensor field $B_{\mu\nu}$. As in [20] we take space-time to be euclidean, and take B to be of rank 26. As shown in [11, 33, 34], this system has many different descriptions with different non-commutativity parameters $\Theta^{\mu\nu}$. These different descriptions are labelled by an anti-symmetric tensor $\Phi_{\mu\nu}$, in terms of which $\Theta^{\mu\nu}$, the open string metric $G_{\mu\nu}$ and the open string coupling constant G_o are given by:

$$\begin{aligned} (G + 2\pi\alpha'\Phi)^{-1} &= -\frac{\Theta}{2\pi\alpha'} + (g + 2\pi\alpha' B)^{-1}\,, \\ G_o^2 &= g_c \left(\frac{\det(G + 2\pi\alpha'\Phi)}{\det(g + 2\pi\alpha' B)}\right)^{1/2}. \end{aligned} \tag{3.1}$$

Here $g_{\mu\nu}$ is the closed string metric and g_c is the closed string coupling constant. One convenient choice of Φ is:

$$\Phi^{(1)} = 0\,, \tag{3.2}$$

with the corresponding non-commutativity parameter $\Theta_{(1)}$ and the open string metric and coupling constants $G_{(1)}$ and $G_{o(1)}$ determined from eq. (3.1). The solutions given in [20] were constructed in these variables. The other choice, which is useful in constructing background independent variables [20, 22], is

$$\Phi^{(2)} = -B\,, \tag{3.3}$$

which gives

$$\Theta_{(2)} = B^{-1}\,, \qquad G_{(2)} = -(2\pi\alpha')^2 B g^{-1} B\,, \qquad G_{o(2)}^2 = g_c \det(2\pi\alpha' B g^{-1})^{1/2}\,. \tag{3.4}$$

We shall denote by $\widehat{A}_\mu^{(1)}$ and $\widehat{A}_\mu^{(2)}$ the non-commutative gauge fields in the first and the second description respectively, and by

$$\widehat{F}^{(s)}_{\mu\nu} = \partial_\mu \widehat{A}^{(s)}_\nu - \partial_\nu \widehat{A}^{(s)}_\mu - i\left[\widehat{A}^{(s)}_\mu, \widehat{A}^{(s)}_\nu\right]_{\Theta_{(s)}}, \qquad s = 1, 2 \tag{3.5}$$

the corresponding non-commutative field strength.[2] Here $[\,,]_{\Theta_{(s)}}$ denotes that we should compute the commutator using the non-commutative product with parameter $\Theta_{(s)}$:

$$[x^\mu, x^\nu]_{\Theta_{(s)}} = i\Theta^{\mu\nu}_{(s)}\,. \tag{3.6}$$

[2] We shall use real coordinate system x^μ instead of the complex one used in [20]. Also, for gauge fields we use the sign convention of ref. [11] which differs from that of ref. [20] by a minus sign.

From now on, we shall drop the subscript $\Theta_{(s)}$ on various commutators. Defining new variables,

$$C_\mu^{(s)} = \left(\Theta_{(s)}^{-1}\right)_{\mu\nu} x^\nu + \widehat{A}_\mu^{(s)}\,, \tag{3.7}$$

we get

$$\widehat{F}_{\mu\nu}^{(s)} = -i\left[C_\mu^{(s)}, C_\nu^{(s)}\right] + (\Theta_{(s)}^{-1})_{\mu\nu}\,. \tag{3.8}$$

The effective action involving the tachyon and the gauge field in the two descriptions is given by [11, 20, 22]:

$$\begin{aligned}
&\frac{\sqrt{\det \Theta_{(s)}}}{G_{o(s)}^2 \alpha'^{13}(2\pi)^{12}} \times \\
&\times \operatorname{Tr}\left[\mathcal{V}(T)\sqrt{\det\left((G_{(s)})_{\mu\nu} + 2\pi\alpha'\Phi_{\mu\nu}^{(s)} - 2\pi\alpha'\left(i\left[C_\mu^{(s)}, C_\nu^{(s)}\right] - (\Theta^{-1})_{\mu\nu}\right)\right)} + \right.\\
&\qquad \left. +\ \alpha' f(T)\left[C_\mu^{(s)}, T\right]\left[T, C_\nu^{(s)}\right](G_{(s)})^{\mu\nu}\sqrt{\det(G_{(s)})} + \cdots\right],
\end{aligned} \tag{3.9}$$

where $\mathcal{V}(T)$ is the tachyon potential which vanishes at $T = T_{\min}$, and $f(T)$ is an unknown function of T. Here Tr denotes trace over infinite-dimensional matrices used to represent various functions of x^μ following the procedure of [12]. As is customary, we shall denote by N the dimension of these matrices, with the understanding that N is actually infinite. $\cdots$ in eq. (3.9) denote various higher derivative terms.

The relationship between the two sets of variables can be found by noting that the ordinary gauge field strength $F_{\mu\nu}$ is related to $\widehat{F}_{\mu\nu}^{(s)}$ through the relation:

$$\widehat{F}^{(s)} = (1 + F\Theta_{(s)})^{-1}F\,, \qquad F = \widehat{F}^{(s)}\left(1 - \Theta_{(s)}\widehat{F}^{(s)}\right)^{-1}, \qquad \text{for} \quad s = 1, 2\,. \tag{3.10}$$

In particular, if $[C_\mu^{(s)}, C_\nu^{(s)}] = 0$, it corresponds to $F = \infty$ for both values of s. Since under an $\mathrm{U}(N)$ gauge transformation

$$C_\mu^{(s)} \longrightarrow U C_\mu^{(s)} U^\dagger\,, \tag{3.11}$$

$C_\mu^{(1)} = 0$ ($C_\mu^{(2)} = 0$) represents $\mathrm{U}(N)$ invariant gauge field configurations [20].

For the choice $s = 2$, one can also define the background independent variables X^μ as follows:

$$X^\mu = (\Theta_{(2)})^{\mu\nu} C_\nu^{(2)}\,. \tag{3.12}$$

The action expressed in terms of these variables takes the form [20, 22]:

$$\begin{aligned}
&\frac{1}{g_c \alpha'^{13}(2\pi)^{12}} \times \\
&\times \operatorname{Tr}\left[\mathcal{V}(T)\sqrt{\det\left(\delta_\mu{}^\nu - 2\pi\alpha' i g_{\mu\rho}[X^\rho, X^\nu]\right)} + \alpha' f(T) g_{\mu\nu}[X^\mu, T][T, X^\nu] + \cdots\right].
\end{aligned} \tag{3.13}$$

Written in these variables, the action does not depend on the choice of the background B field.

Denoting by I_m the $m \times m$ identity matrix for any integer m, the tachyonic vacuum corresponds to $T = T_{\rm min} I_N$, and $\mathcal{V}(T)$ vanishes in this vacuum. This implies that the kinetic term for the $C_\mu^{(s)}$ fields vanishes at $T = T_{\rm min} I_N$ and hence the $C_\mu^{(s)}$'s themselves are not good coordinates around this point.[3] Instead we should choose new variables

$$\widetilde{C}_\mu = h_\mu(T, \vec{C})\,, \qquad \widetilde{T} = h(T, \vec{C}) \tag{3.14}$$

to describe the field configuration around this point. (Note that we have dropped the superscript (s) from C, — the discussion below holds for either choice of s, and also for $\vec{C}$ replaced by the background independent variables $\vec{X}$. The precise form of the functions h_μ and h will of course depend on whether we choose $C_\mu^{(1)}$, $C_\mu^{(2)}$ or X^μ in the arguments of these functions.) Vanishing of the C_μ kinetic term at $T = T_{\rm min} I_N$ suggests that the functions h_μ and h have the property:[4,5]

$$h_\mu(T_{\rm min} I_N, \vec{C}) = 0\,, \qquad h(T_{\rm min} I_N, \vec{C}) = \widetilde{T}_{\rm min} I_N\,, \tag{3.15}$$

where $\widetilde{T}_{\rm min}$ is some constant which could also be set to zero by redefining $\widetilde{T}$. We shall assume that the functions h_μ and h appearing in eq. (3.14) may be expressed as (infinite) sum of products of various powers (possibly fractional) of T and C_μ, so that eqs. (3.15) hold as operator equations irrespective of the dimension of the operators. In that case, for any non-singular matrix S, we have

$$\begin{aligned} h_\mu(STS^{-1}, S\vec{C}S^{-1}) &= S h_\mu(T, \vec{C}) S^{-1}\,, \\ h(STS^{-1}, S\vec{C}S^{-1}) &= S h(T, \vec{C}) S^{-1}\,. \end{aligned} \tag{3.16}$$

In particular, if we take S to be an $N \times N$ unitary matrix, then the above relation shows that $\widetilde{T}$ and $\widetilde{C}_\mu$ defined in eq. (3.14) transform in the adjoint representation of $\mathrm{U}(N)$. Thus we can get an $\mathrm{U}(N)$ invariant configuration by taking:

$$\widetilde{T} = \widetilde{T}_{\rm min} I_N\,, \qquad \widetilde{C}_\mu = 0\,. \tag{3.17}$$

We identify this as the "nothing" state. Note however, that this does not require us to take $C_\mu = 0$; any finite C_μ corresponds to this vacuum when $T = T_{\rm min} I_N$. Thus *the coordinate redefinition suggested in eqs. (3.14) and (3.15) gets rid of the problem of having degenerate vacua labelled by different values of the gauge field vacuum expectation values.*

[3] As we lack sufficient intuition about theory of matrices, it is hard to conclude just by looking at eqs. (3.9) or (3.13) that the variables appearing in these equations are wrong variables. Here we are using our intuition from conventional field theories; — since when expressed in the space-time language the kinetic term for the gauge fields vanish, different gauge field configurations, labelled by different values of C_μ (X^μ), should correspond to same physical configurations at $T = T_{\rm min} I_N$.

[4] The important point here is that $h_\mu(T_{\rm min} I_N, \vec{C})$ and $h(T_{\rm min} I_N, \vec{C})$ should be independent of $\vec{C}$. A constant shift in the definition of $\widetilde{C}_\mu$ can then be used to set $h_\mu(T_{\rm min} I_N, \vec{C})$ to zero.

[5] For our purpose it will be sufficient if these relations hold for those C_μ's which satisfy equations of motion.

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The direct evidence for eq. (3.15) comes from examining eq. (3.14) with $\vec{C}$ replaced by $\vec{X}$. If we consider a configuration of commuting X^μ's, then the eigenvalues of X^μ can be regarded as the positions of D-instantons making up the original D25-brane [35, 36, 22]. Now we can invoke the string field theory analysis of ref. [26, 27] to argue that when the tachyon rolls down to its minimum $T_{\rm min}$, there is no flat direction of the potential, and hence all the different (commuting) values of X^μ must correspond to the same point in the configuration space. Equations (3.14) and (3.15) (with $\vec{C}$ replaced by $\vec{X}$) clearly incorporates this.

Let us now turn to the soliton solutions. Expressed in terms of the original variables T, C_μ, these solutions are typically of the block diagonal form [20]:

$$T = \begin{pmatrix} T_{\rm max} I_M & \\ & T_{\rm min} I_{N-M} \end{pmatrix}, \qquad C_\mu = \begin{pmatrix} S_\mu & \\ & V_\mu \end{pmatrix}, \tag{3.18}$$

where $T_{\rm max}$ is the value of the tachyon at which the potential has a local maximum, representing the original D-brane before tachyon condensation, M is an integer (possibly infinite) and S_μ and V_μ are $M\times M$ and $(N-M)\times(N-M)$ matrices, respectively. The unwanted degeneracy of the solutions comes from different choices of V_μ. S_μ's on the other hand represent world-volume fields on the resulting solitonic brane, and different values of S_μ correspond to different background field configurations on this brane.

Using (3.14), and the assumption that h_μ and h involve sum of products of various powers of T and C_μ, the solution (3.18) can be rewritten as

$$\begin{aligned} \widetilde{C}_\mu &= \begin{pmatrix} h_\mu(T_{\rm max} I_M, \vec{S}) & \\ & h_\mu(T_{\rm min} I_{N-M}, \vec{V}) \end{pmatrix}, \\ \widetilde{T} &= \begin{pmatrix} h(T_{\rm max} I_M, \vec{S}) & \\ & h(T_{\rm min} I_{N-M}, \vec{V}) \end{pmatrix}. \end{aligned} \tag{3.19}$$

Using eq. (3.15), and the fact that it represents operator relations irrespective of the dimension of the operators, we can rewrite eq. (3.19) as

$$\widetilde{C}_\mu = \begin{pmatrix} h_\mu(T_{\rm max} I_M, \vec{S}) & \\ & 0 \end{pmatrix}, \qquad \widetilde{T} = \begin{pmatrix} h(T_{\rm max} I_M, \vec{S}) & \\ & \widetilde{T}_{\rm min} I_{N-M} \end{pmatrix}. \tag{3.20}$$

From this we see that in terms of the new variables solutions with different background values of $\vec{V}$ correspond to the same field configuration. Thus *the apparent unwanted degeneracy of the soliton solutions disappears when we make the right choice of variables in describing these solutions.*

Since different background values of V_μ correspond to the same tachyonic soliton configuration, we can choose any of these representative values to analyse the tachyon condensation problem. As was shown in refs. [13, 14, 20], many exact results can be obtained by taking $\vec{V}$ to be zero, which corresponds to taking the value of the

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ordinary magnetic field strength away from the location of the soliton to infinity. Thus if our proposal, — that the correct choice of coordinates is given by $\widetilde{C}_\mu$ and $\widetilde{T}$ defined in eqs. (3.14) and (3.15) — is correct, then the exact results for $\vec{V} = 0$ can be used to conclude that the same results also hold for $\vec{V} \neq 0$. Translated to conventional language, this will imply that the exact results, obtained in the limit of large asymptotic magnetic field on the original D-25 brane, are also valid when the asymptotic magnetic field is finite.[6] For magnetic field in the plane transverse to the soliton this is a surprising result, since for finite asymptotic magnetic field higher derivative terms in the effective action can no longer be ignored in the analysis, and the shape of the soliton is expected to change. But if eqs. (3.14) and (3.15) are correct, then this must be a gauge artifact.

An indirect evidence for this follows from the fact that the D23-brane, after all, is described by a specific conformal field theory, and hence must be described by a unique configuration in string theory. Thus any two different configurations, describing the D23-brane in the same closed string background, must be gauge equivalent.

4. The fate of the bifundamental states

The analysis of refs. [14, 20] found the existence of certain finite mass open string states on the world-volume of the soliton which transform in the bifundamental representation of $\mathrm{U}(M) \times U(N-M)$. These arise from fluctuations of off block-diagonal components of C_μ and T around the background (3.19). These states are not present in the excitation spectrum of a lower-dimensional D-brane. Hence the existence of these states on the soliton world-volume poses a problem for identifying the solitons as lower-dimensional D-branes.

Physically, these modes represent open string states with one end on the D-brane soliton and the other end on the tachyonic vacuum [15]. Since open strings cannot end on the vacuum, these states must be absent from the spectrum. A possible explanation for the absence of these states was given in [29] (for related but alternative explanations, see refs. [30, 31]). The main point here is that since the kinetic term for the U(1) gauge field living in the vacuum vanishes (we are using the original variables before the field redefinition), the equations of motion of the gauge field will force the currents coupled to the gauge field to vanish. These constraints will remove from the spectrum the states which are charged under the gauge field. This includes the open string states with one end on the D-brane soliton and the other end in the tachyonic vacuum.

The existence of these constraints can be seen explicitly in the present framework. Since we have argued that backgrounds labelled by different choices of V_μ in eq. (3.18) correspond to physically identical configurations, we shall choose a background where

[6] A similar remark was made in ref. [14].

V_μ are non vanishing, giving finite asymptotic magnetic field. We can now consider fluctuating fields of the form:

$$T = \begin{pmatrix} T_{\max} I_M & \\ & T_{\min} I_{N-M} \end{pmatrix} + \begin{pmatrix} \widehat{S} & \widehat{W} \\ \widehat{W}^\dagger & \widehat{V} \end{pmatrix},$$
$$C_\mu = \begin{pmatrix} S_\mu & \\ & V_\mu \end{pmatrix} + \begin{pmatrix} \widehat{S}_\mu & \widehat{W}_\mu \\ \widehat{W}_\mu^\dagger & \widehat{V}_\mu \end{pmatrix}, \qquad (4.1)$$

where the hatted variables correspond to fluctuations around the background (3.18). In particular, the fluctuations $\widehat{V}_\mu$ represent fluctuations of the gauge fields in the vacuum *outside the soliton*. Since $\mathcal{V}(T_{\min} I_{N-M}) = 0$, we see from eq. (3.9) that the regular kinetic term for $\widehat{V}_\mu$ (which would have been proportional to $\mathrm{Tr}[V_\mu, \widehat{V}_\nu][V^\mu, \widehat{V}^\nu]$ had $\mathcal{V}(T_{\min} I_{N-M})$ been finite) vanishes in the background (3.18). Thus the equations of motion of $\widehat{V}_\mu$ does not contain any term linear in $\widehat{V}_\mu$, and hence, instead of determining the gauge fields in terms of the currents, imposes constraints containing quadratic and higher powers of the bifundamental fields $\widehat{W}$, $\widehat{W}_\mu$. These constrains ensure absence of states charged under $\widehat{V}_\mu$.

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